Quick Clicker Question:

Solve $x^2 + 1 = 0$.

(A) Silly, you can't do that!
(B) $x = 1$
(C) $x = -1$
(D) $x = \sqrt{-1}$
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7. (C) $x = -1$
8. (D) $x = \sqrt{-1}$
When the fisherman, let's call him Jeremy, comes in with many fish, that is good. When Jeremy comes in with not many fish, that is not good. Jeremy likes good. His evening with his wife and extended family is MUCH nicer after a good catch. Thus began the branch of philosophy called goodosophy (or ethics)! But Jeremy is also credited with fathering the branch of philosophy that is today called mathematics...
Counting Fish

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1 fish, 2 fish,
Jeremy, being a fisherman of deep thought, realized he could count his catch and fish longer if he needed, in order to have a good evening.

Hence, the counting numbers!

1, 2, 3, 4, 5, 6, 7, 8, 9,...

Two things here:

First, Jeremy and his kin didn't yet have Arabic numerals (didn't even count in base 10?).

Second, much later in India, the idea of 0...
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If she makes her customers happy, that is good.
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Sophie realized that 2 customers arguing over the last fish could both be made happy by cutting it in half (lengthwise, of course, and charging a bit more for the effort (thereby employing her son-in-law)).
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... for why else would anyone need to solve $3x = 17$ for $x$, when there is a butcher around?
Sophie’s great, great, ..., granddaughter, Sophia, is a tax collector of deep thought.
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\[
\text{If she can “balance the books”, that is good.}
\]

\[
\text{If she cannot account for all acreage, that is not good.}
\]

Sophia knows that the area of a square with side \( s \) is \( \text{area} = s^2 \).

To balance the books, she needs to find \( s \) ? Hence, the square root.

\[
\begin{align*}
\text{area} & = 1, \\
\text{area} & = 4, \\
\text{area} & = 9, \\
\text{area} & = 16, \\
\text{area} & = 25, \\
\text{area} & = 36, \ldots
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$s^2 = 1, \ s^2 = 4, \ s^2 = 9, \ s^2 = 16, \ s^2 = 25, \ s^2 = 36,...$
But what about \( s^2 = 2 \)? (or \( s^2 = 3 \), or \( s^2 = 5 \), etc)?
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Because she showed that if \( s^2 = 2 \), then \( s \) cannot be a ratio!
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He said $x = \sqrt{-1}$. And he is pissed because his teacher (who seems to like his girlfriend!) whacked his knuckles for getting it “wrong”...