Quick Questions:

Clicker Question:
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Clicker Question: Do you know you have to bring your clicker every day?
   (A) Yes       (B) No.
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   (A) Yes     (B) No.
Let’s talk about the grading scheme....
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Stay on topic. But use the other areas of the forum too! There’s a place for anything you want to talk about.
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But that same argument means 3/4, 6/8 are different quantities! Are they? How about .75, .750, .7500, etc? Are these all different too?
On the DD, people seemed to confuse

.9999

with

.9999....
On the DD, people seemed to confuse

\[ .9999 \]

with

\[ .9999\ldots \]

Those little dots make a *huge* difference. \( .9999 \) is not at all the same as \( 1.0000 \)

\[ 1.0000 - .9999 = .0001 \]
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with

.9999...

Those little dots make a huge difference. .9999 is not at all the same as 1.0000

1.0000 − .9999 = .0001

But .9999.... is debatable:

1.000... − .9999... = .00000...
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\[ 1.0000 - .9999 = .0001 \]

But .9999.... is debatable:

\[ 1.000... - .9999... = .00000... \]

*Mathematicians are incredible sticklers for precision—we have to be!!*
Most of the debate on the forum revolved around the meaning of

\[ 1.000... - 0.9999... = 0.0000... \]

Is there anything left at the end of the “0”s
Humpty Dumpty had the right idea, in *Alice’s Adventures Through The Looking Glass*

(Not coincidentally at all, Lewis Carroll was the pen name of a logician and mathematician, Charles Dodgeson.)
‘I don’t know what you mean by “glory”,’ Alice said.

Humpty Dumpty smiled contemptuously. “Of course you don’t – till I tell you. I meant “there’s a nice knock-down argument for you!”’

‘But “glory” doesn’t mean "a nice knock-down argument",’ Alice objected.
‘When I use a word,’ Humpty Dumpty said, in rather a scornful tone, ‘it means just what I choose it to mean – neither more nor less.’

‘The question is,’ said Alice, ‘whether you can make words mean so many different things.’
‘The question is,’ said Humpty Dumpty,

‘which is to be master – that is all.’
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‘which is to be master – that is all.’

The history of “number”, largely, is a history of refining our meaning, becoming masters of our intentions.
There is nothing "real" about the real numbers. In no real sense can you find numbers with infinite precision:
1.221929291992948448212... _forever and ever_
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and yet ... real numbers are intensely useful.
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and yet ... real numbers are intensely useful. In just the same way, there are no perfect circles in the real world, yet the idea of a perfect circle explains a lot of what we see around us.
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The best answer to our question about .999…. is: which idea has more utility?
The mathematician Kronecker said: “God made the integers and man made the rest.”
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That is, he meant that the counting numbers are part of reality, but the rest is our *interpretation* of reality, or a *tool* for thinking about reality.
It’s not even clear to me that the counting numbers are necessarily “real”!
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How many rocks do you see?
It’s not even clear to me that the counting numbers are necessarily “real”!
How many rocks do you see? Where does one rock begin and another end?
It’s not even clear to me that the counting numbers are necessarily “real”!
How many aspens in the grove in the foreground?
It’s not even clear to me that the counting numbers are necessarily “real”!
How many aspens in the grove in the foreground? Is this one organism or hundreds?
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How many aspens in the grove in the foreground? Is this one organism or hundreds?
It’s not even clear to me that the counting numbers are necessarily “real”!
How many organisms the Great Barrier Reef?
Ok, let’s at least assume we can **Count!** Where do we go from there?
The ancient greeks conceived of numbers as *ratios*, as proportions. You could have one of something,
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or twice as much (2) 
or three times as much (3).
The ancient greeks conceived of numbers as ratios, as proportions. You could have one of something, or twice as much (2), or three times as much (3). You could have half of that.
The ancient greeks conceived of numbers as *ratios*, as proportions. You could have one of something,

or twice as much (2)

or three times as much (3).

You could have half of that

or even one half of three (3/2).
Filling in all possible ratios, you can get any quantity you like: 17/32 or 102391/329912, filling in every single spot on the number line, like the marks on a very fine ruler.
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Or can you?
Filling in all possible ratios, you can get any quantity you like: 17/32 or 102391/329912, filling in every single spot on the number line, like the marks on a very fine ruler.

*Or can you? Does this get everything?*
Famously, Hippasus of Metapontum was murdered for discovering a proof that the length of a diagonal of a square (that is, $\sqrt{2}$) cannot be a ratio of two whole numbers!
But then what other kinds of quantities are possible, of not all numbers are *rational*? Today we talk about *real* numbers— a continuum of quantities, filling in all the gaps that seem to be missing in the rational numbers.

But, really, *what’s so real about them*?
Here’s a simple question:
Is $0.99999999\ldots = 1.000000\ldots$?
Here’s a simple question:

Is $0.99999999\ldots = 1.000000\ldots$?

You might know what the answer is *supposed* to be, but what do you really think?

(A) Yes    (B) No    (C) Depends